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CRIMINAL CHOICE, NONMONETARY SANCTIONS, AND MARGINAL
DETERRENCE: A NORMATIVE ANALYSIS

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ABSTRACT

This paper develops a normative model of optimal sanctions in the Becker Tradition which emphasizes the role of marginal deterrence. The paper complements Shavell's 1987 *American Economic Review* paper, the essential difference being that Shavell's model concentrates on variations in the sanction imposed within a single category of acts (a specific crime) while the model in this paper concentrates on variations in the sanction imposed across categories of acts (different crimes). In their most general formulations, neither Shavell's model nor the model developed in this paper yields the result that acts with greater social harm should receive greater sanctions. But special cases, which readers may or may not find reasonable, do yield that result, within crimes for both models and across crimes in the model developed in this paper. This paper also identifies the necessary condition of jointness in the cost of law enforcement in the case of comparisons across crimes.

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Normative models of the optimal use of sanctions, monetary as well as nonmonetary, that employ the assumption of precommitment on the part of social authorities provide an important and useful benchmark for the evaluation of existing policies. Ever since Gary Becker published his classic article on the economics of crime (Becker, 1968), however, a conundrum has plagued the literature: if law enforcement is costly but crimes are socially undesirable and potentially deterrable, then efficiency requires that for all crimes the probability of apprehension be set arbitrarily low and the sanction arbitrarily high (see, e.g., Carr-Hill and Stern, 1979). This solution imposes no costs on society as long as the expected sanction is high enough to deter all crime; since no crime is ever committed, the sanction never need be imposed. Hence, even if sanctions are costly to impose—e.g. in the case of nonmonetary sanctions—this is the efficient solution.

Two early attempts in the modern literature to find an explanation for nontrivial criminal sanctions (i.e., sanctions which vary systematically with such factors as the level of harm) were offered by Becker in his original article and, later, George Stigler (1970). Becker proposed including in the social calculus the value of the gain to offenders from engaging in proscribed behavior. Stigler re-introduced the notion of marginal deterrence to the modern literature. The idea of marginal deterrence is quite simple in principle—if an offender is to be punished the same for, say, armed robbery and murder, there is no incentive not to commit murder during an armed robbery. At the margin, there is no deterrent to inflicting on society the additional harm associated with murder given that armed robbery has already been committed.

The purpose of this paper is to show that marginal deterrence is neither necessary nor sufficient for nontrivial criminal sanctions. In the process, I will consider several specific examples, and will, for these examples, identify when marginal deterrence does play a role.

The basic formulation of my model follows closely that of Shavell (1987). In particular, social welfare is defined to be the social value of the private benefits obtained from engaging in proscribed behavior, minus the harm inflicted, the costs of apprehension and the costs of imposing sanctions. The social authorities choose sanctions and an allocation of resources to apprehension so as to maximize social welfare. I also assume the existence of nondeterrables, which requires that sanctions be bounded above. The new element in my model is a richer set of proscribed behaviors from which potential offenders choose their actions and, consequently, a more complex set of relationships between resources allocated to apprehension and probabilities of apprehension.

In section II, I specify a general two-crimes model, and demonstrate that the socially optimal set of sanctions and allocation of resources to apprehension typically requires different sanctions for each crime, one of which must equal the maximum possible sanction. When the allocation of resources is fixed, the optimal sanction associated with the other crime is always increasing in the level of harm associated with that crime and decreasing in the level of harm associated with the first crime. When the allocation of resources to apprehension is allowed to adjust optimally in response to changes in harm levels, however, the relationships between optimal sanctions and changes in harm levels become ambiguous in general.

In section III, I show that in my model a nontrivial optimal sanctions policy depends crucially on a notion of nonseparability in the costs of apprehension. In particular, if the probabilities of apprehension for the two crimes can be controlled separately, and the total allocation of resources to apprehension is simply the sum of resources allocated to such crime, then the optimal sanction for each crime will be the maximum possible sanction. The reasoning is, of course, well known, and is essentially the same as the reasoning behind the Becker conundrum.

In section IV, I consider a special case that focusses on a classic example of the need for marginal deterrence. In particular, the general two-crimes model is applied to the case of armed robbery and murder in the commission of armed robbery. Private benefits are assumed to be the same for either act, but social harm is greater for the latter and the probability of apprehension lower (because, say, the murder victim is a witness). In this case the optimal sanction for armed robbery is always less than the optimal sanction for murder in the commission of armed robbery, the latter which equals the maximum possible sanction. As the harm associated with armed robbery increases, the overall allocation of resources to apprehension increases, but the optimal sanction for murder in the commission of armed robbery increases if and only if a natural condition on the apprehension technology is satisfied.

In section V, I consider another example in which it might be expected that marginal deterrence would play a role in the optimal enforcement and sanctions policy, simple assault versus aggravated assault. Private benefits for aggravated assault are assumed to be greater than for simple assault by a constant factor for all potential offenders. Social harm also is assumed to be greater for aggravated assault than for simple assault but the probability of apprehension the same. In this case the optimal sanction for aggravated assault is always the maximum possible sanction. The optimal sanction for simple assault will be strictly less than this if and only if social harm levels differ by at least the same factor as private benefits; otherwise marginal deterrence plays no role.

In section VI, I briefly compare these results to the existing literature, and offer some observations regarding the implications of the results and possible future work.

II. THE MODEL

In this model potential offenders can choose to commit one of two proscribed acts or neither, but not both. Thus the following variables are defined for $i = 1, 2$:¹

- b_i = private benefits from a harmful act,
- h_i = harm caused by the act,
- p_i = probability of apprehension,
- s_i = sanction,

σ = proportionality factor in cost of imposing sanctions, and
 r = total resources allocated to apprehension.

Assumption 1: $s \in [0, \bar{s}]$: sanctions are bounded above by \bar{s} .

Assumption 2: $(b_1, b_2) \sim f(b_1, b_2)$: private benefits are distributed via joint density $f(\cdot, \cdot)$.

Assumption 3: $p_i = g_i(r)$, where $g_i(0) = 0$, $g_i' > 0$ and $g_i'' < 0$ for $i = 1, 2$.

Assumption 1 follows Shavell (1985, 1987). Assumption 2 is obvious, but Assumption 3 requires comment. Fundamentally it reflects a kind of nonseparability, or jointness, in the production of criminal apprehension rates. It presumes that if a given amount of resources are devoted to law enforcement, then some of these resources are allocated to fixed costs which must be incurred regardless of which proscribed behavior society attempts to control. In this case, $g_1(\cdot)$ and $g_2(\cdot)$ are "reduced form" apprehension rates which come from a maximizing process associated with optimal behavior by the law enforcement agency. In this paper I take these apprehension rates as given, although an obvious line of research is to make them endogenous.

Assumption 4: $h_1 < h_2$: social harm for crime 1 is less than crime 2.

Assumption 4 is really just a matter of notation, although it provides a convenient way to resolve indifference. In particular, for given probabilities of apprehension and sanction levels, since potential offenders are restricted to committing one crime or none, they behave according to the following rule:

$$\text{if } \begin{cases} b_1 \leq p_1 s_1 \text{ and } b_2 \leq p_2 s_2 \\ b_1 > p_1 s_1 \text{ and } b_2 \leq b_1 - p_1 s_1 + p_2 s_2 \\ b_2 > p_2 s_2 \text{ and } b_1 < b_2 - p_2 s_2 + p_1 s_1 \end{cases} \text{ then } \begin{cases} \text{commit no crime} \\ \text{commit crime 1} \\ \text{commit crime 2} \end{cases} . \quad (1)$$

In (1), indifference is resolved in favor of the crime with the lowest social harm (crime 1).

Social welfare is defined to be the social value of the private benefits obtained from engaging in proscribed behavior minus the harm inflicted, the costs of imposing sanctions and the costs of apprehension; i.e.,

$$\begin{aligned} W(s_1, s_2, r) = & \int_{p_1 s_1}^{\infty} \int_0^{b_1 - p_1 s_1 + p_2 s_2} (b_1 - h_1 - \sigma p_1 s_1) f(b_1, b_2) db_2 db_1 \\ & + \int_{p_2 s_2}^{\infty} \int_0^{b_2 - p_2 s_2 + p_1 s_1} (b_2 - h_2 - \sigma p_2 s_2) f(b_1, b_2) db_1 db_2 - r, \end{aligned} \quad (2)$$

where $p_1 = g_1(r)$ and $p_2 = g_2(r)$. The social problem is to choose sanction levels and an allocation of resources to apprehension so as to maximize $W(s_1, s_2, r)$. The optimal values are denoted s_1^* , s_2^* , and r^* .

Proposition 1: Given Assumptions 1–3, if $r^* > 0$ then optimal sanctions have the following properties:²

- (a) At least one optimal sanction must equal \bar{s} . The other optimal sanction may equal \bar{s} or it may be strictly less than \bar{s} .
- (b) If the allocation of resources to apprehension is fixed and the optimal sanction associated with a crime is less than \bar{s} , then it is increasing in the level of harm associated with that crime and decreasing in the level of harm associated with the other crime.
- (c) If the allocation of resources to apprehension is allowed to adjust optimally to changes in harm levels, and the optimal sanction associated with a crime is less than \bar{s} , then the relationship between it and either harm level is ambiguous in general.

The formal proof of this proposition is given in the appendix, but it can be explained easily. Suppose both sanctions are less than the maximum feasible sanction. Inspection of $W(s_1, s_2, r)$ reveals that social welfare only depends on expected sanctions, $p_1 s_1$ and $p_2 s_2$; nowhere do s_1 or s_2 appear in any other form than multiplicatively with p_1 and p_2 respectively. Meanwhile, if $r = 0$, sanctions are irrelevant. Thus it must be that $r > 0$, and consequently so are p_1 and p_2 . In this case, however, since s_1 and s_2 are less than \bar{s} , it is possible to lower r and increase both s_1 and s_2 but hold $p_1 s_1$ and $p_2 s_2$ constant. This must increase social welfare. Eventually either s_1 or s_2 hits \bar{s} , the maximum feasible sanction. Further decreases in r will increase the remaining sanction, until either a balance between marginal social gains and losses has been achieved with the remaining sanction less than \bar{s} , or after it too hits \bar{s} . In the latter case expected sanctions will still differ across the two crimes, but only due to differences in the probability of apprehension; the sanction imposed given apprehension will be the maximum feasible sanction for either crime. In other words, marginal deterrence will not play a role.

The comparative statics results with respect to harm levels are less obvious, but not unintuitive. If resources allocated to apprehension are held constant, an increase in the harm associated with a crime will increase the optimal sanction associated with it so long as the increase in harm *increases* the marginal gain from higher sanctions on the crime; e.g., for crime i , say, so long as W_{s_i, h_i} is positive. This will always be the case, however, since the higher the harm associated with a crime, the greater the social gain from deterring it. On the other hand, if one considers s_i^* , say, and lets h_j increase (where $i \neq j$), then s_i^* will decrease so long as W_{s_i, h_j} is negative. This, again, will always be the case since the higher the harm associated with crime j , the more society would prefer potential offenders to commit crime i and thus the lower the marginal social gain to increases in s_i .

Finally, these comparative statics results become ambiguous when the allocation of resources to apprehension is allowed to adjust optimally to changes in harm levels. As is shown in

the appendix, a sufficient condition for $ds_i^*/dh_i > 0$ and $ds_i^*/dh_j < 0$ is $W_{s_i,r} < 0$ (for $i = 1, 2$ and $j \neq i$). But this condition has no obvious economic interpretation in the general model.

III. JOINTNESS IN THE PRODUCTION OF LAW ENFORCEMENT

The model of section II has three central elements: (1) the existence of nondeterrables (and the boundedness of sanctions); (2) the need for marginal deterrence (the consideration of a spectrum of crimes and potential offenders' choices of an optimal crime, or no crime, from this spectrum); and (3) jointness in the production of law enforcement. In this section I demonstrate first that (3) is necessary.

Assumption 3': $p_1 = g_1(r_1)$, $p_2 = g_2(r_2)$ and $r = r_1 + r_2$, where $g_i(0) = 0$, $g_i' > 0$ and $g_i'' < 0$ for $i = 1, 2$.

Proposition 2: Given Assumptions 1, 2, and 3', if $r_1^* > 0$ and $r_2^* > 0$ then optimal sanctions have the following properties:³

- (a) Both optimal sanctions must equal \bar{s} .
- (b) If the allocation of resources to apprehension is fixed and the optimal sanction associated with a crime is less than \bar{s} , then it is increasing in the level of harm associated with that crime and decreasing in the level of harm associated with the other crime.

The formal proof of this proposition is again given in the appendix, but the logic behind the result is similar to that behind Proposition 1. The essence of Assumption 3' is that p_1 and p_2 can be set independently. Social welfare still only depends on expected sanctions though. Thus, if either s_1 or s_2 is less than \bar{s} , social welfare can be increased by lowering r_1 or r_2 (and thus p_1 or p_2) and raising s_1 or s_2 so as to hold $p_1 s_1$ and $p_2 s_2$ constant. In this way, a constant level of deterrence is achieved at a lower social cost in terms of resources allocated to apprehension. If, however, r_1 and r_2 are fixed in such a way that either s_1^* or s_2^* is less than \bar{s} , then the comparative statics of changes in h_1 or h_2 are identical to those given in Proposition 1, part b.⁴

To be fair, there is a kind of marginal deterrence present in the optimal policy even when enforcement costs are separable in the sense of Assumption 3'. Actual sanctions for both crimes will be \bar{s} , but the optimal expected sanctions, $p_1^* \bar{s}$ and $p_2^* \bar{s}$, will differ. It is still the case, however, that all offenders who are apprehended, regardless of their crime, will receive the maximum possible sanction.

IV. MARGINAL DETERRENCE: Example 1

In this section I consider the following classic example of the need for marginal deterrence in an optimal enforcement and sanctions policy. Suppose a potential offender chooses between two crimes, (1) armed robbery and (2) murder in the commission of armed robbery. Assume the benefit to the offender is the same for either crime, but that killing a witness (for example the victim of the robbery) reduces the probability of apprehension. Assume also that murder in the commission of armed robbery generates greater social harm than armed robbery alone. The next proposition shows

that under these assumptions, murder in the commission of an armed robbery should receive the greater sanction (equal to the maximum feasible sanction) and that armed robbery should receive a lesser sanction. Furthermore, the latter sanction will increase in the level of harm associated with armed robbery if and only if a natural condition on g_1 and g_2 holds.

Proposition 3: Suppose, given Assumptions 1, 3 and 4, that $h_1 < h_2$, $g_1(r) > g_2(r)$ for all $r > 0$ and $b_1 = b = b_2$. Let b be distributed via the density $f(b)$. If $r^* > 0$ then optimal sanctions have the following properties:

- (a) The optimal sanction for murder in the commission of armed robbery equals \bar{s} . The optimal sanction for armed robbery is strictly less than \bar{s} .
- (b) The sanction for armed robbery increases in the harm associated with armed robbery if and only if $g_2(r)/g_1(r)$ is increasing in r .

Under the hypotheses of Proposition 3, suppose $s_1^* = \bar{s}$. In this case, all potential offenders must prefer crime 2 to crime 1 since the sanction on it cannot be higher than the sanction on crime 1 and the probability of apprehension is lower. The logic of the Becker conundrum then implies $s_2^* = \bar{s}$ as well. But if $s_2^* = \bar{s}$, then for a fixed allocation of resources to apprehension, the social authorities can lower s_1^* until potential offenders are indifferent to committing either crime. Since indifference is resolved in favor of the less harmful crime, this will make society better off. Hence if $s_2^* = \bar{s}$, it must be that $s_1^* < \bar{s}$. One sanction must equal \bar{s} , though, so the optimal structure of sanctions requires $s_1^* < \bar{s} = s_2^*$. In fact, $s_1^* = g_2(r^*)\bar{s}/g_1(r^*)$: expected sanctions must be equal given indifference between the crimes and equal private benefits.

Once it is established that $s_1^* = g_2(r^*)\bar{s}/g_1(r^*)$, part (b) of the proposition follows directly. In general, the optimal allocation of resources to apprehension always increases in harm levels. Hence $dr^*/dh_1 > 0$. Whether the sanction on crime 1 increases in the harm level associated with it thus depends entirely on how increases in r^* affect $g_2(r^*)$ versus $g_1(r^*)$ —the "apprehension gap." If an increase in r^* *increases* the proportional apprehension gap then s_1^* must *fall* when h_1 rises to keep potential offenders indifferent to the two crimes. If, as is more natural, an increase in r^* *decreases* the proportional apprehension gap, then s_1^* must *rise* as the pressure for marginal deterrence decreases.

In some respects, this example of the role of marginal deterrence in an optimal enforcement and sanctions policy preserves some of the desirable but unrealistic attributes of the original Becker/Stigler formulation. Under the optimal policy the maximum possible sanction is applied to a crime which never occurs. Thus society never bears the costs of imposing it. At the same time, the crime which *is* committed by some offenders receives a lesser sanction which varies positively with the degree of social harm inflicted by that crime if $g_2(r)/g_1(r)$ is decreasing in r .

V. MARGINAL DETERRENCE: Example 2

A second example of the need for marginal deterrence in an optimal enforcement and sanctions policy involves crimes where the private benefits to offenders differ in a systematic way, but where differential apprehension rates are not an issue. For example, suppose a potential offender

is contemplating simple assault (crime 1) versus aggravated assault (crime 2). The probability of apprehension is approximately the same in either event, but both the private benefit and harm associated with the latter is greater than the former. In particular, let $b_2 = \alpha b_1$ where $\alpha > 1$ so that for all potential offenders the private benefit of aggravated assault is greater than the private benefit for simple assault by a factor of $(\alpha - 1)$. The next proposition shows that under these assumptions, aggravated assault should receive the greater sanction (equal to the maximum feasible sanction) and that simple assault should receive the same sanction or, if a natural necessary and sufficient condition is satisfied, a lesser sanction.

Proposition 4: Suppose, given Assumptions 1, 3, and 4, that $g_1(r) = g_2(r)$ for all $r > 0$ and $b_2 = \alpha b_1$ where $\alpha > 1$. Let b_1 be distributed via density $f(b_1)$. If $r^* > 0$, then optimal sanctions have the following properties:

- (a) The optimal sanction for aggravated assault (crime 2) equals \bar{s} .
- (b) The optimal sanction for simple assault (crime 1) is equal to or less than \bar{s} . A necessary and sufficient condition for $s_1^* < \bar{s}$ is $\alpha h_1 < h_2$. In this case $s_1^* < \bar{s}/\alpha$.

The point of this example is to illustrate a case in which marginal deterrence may not be part of an optimal enforcement and sanctions policy. If $\alpha h_1 > h_2$, then any $s_1 \in [\bar{s}/\alpha, \bar{s}]$ is optimal but crime 1 is never committed. Only crime 2, which has the greater harm is committed. True marginal deterrence only plays a role if the difference in harm levels is high enough. In this case both crimes are committed, simple assault by offenders with $b_1 \in [ps_1, p(\bar{s} - s_1)/(\alpha - 1)]$ and aggravated assault by offenders with $b_1 \in [p(\bar{s} - s_1)/(\alpha - 1), \infty]$.

VI. CONCLUSION

The purpose of this paper has been to develop a normative model of optimal sanctions in the Becker Tradition which emphasizes the role of marginal deterrence. The paper complements Shavell's 1987 *American Economic Review* paper, the essential difference being that Shavell's model concentrates on variations in the sanction imposed within a single category of acts (a specific crime) while the model in this paper concentrates on variations in the sanction imposed across categories of acts (different crimes). The distinction is not just semantic, though; it is driven by whether potential offenders can choose the private benefit and social harm associated with various proscribed behaviors. Moreover, in the absence of allowance for some choice, any theory of optimal sanctions will require some offenders to receive the maximum possible sanction for every category of crime.

In their most general formulations, neither Shavell's model nor the model developed in this paper yields the result that acts with greater social harm should receive greater sanctions. But special cases, which readers may or may not find reasonable, do yield that result, within crimes for both models and across crimes in the model developed in this paper. This paper has also identified the necessary condition of jointness in the cost of law enforcement in the case of comparisons across crimes.⁵

It is important to recognize that all of the results obtained in this paper apply to monetary sanctions as well as nonmonetary sanctions. It may, however, be possible to obtain additional results

for monetary sanctions since in the monetary sanctions model $\sigma = 0$. Shavell (1989b), in particular, compares monetary sanctions between a single-crime model and a two-crimes model, a comparison which is difficult for nonmonetary sanctions.

Despite the rich set of results yielded by the simple model specified in this paper, one must be careful to recognize their limitations. As mentioned in the introduction, normative models in the Becker Tradition are important and useful as a *benchmark* for the evaluation of existing policies. They have less value as an explanation of existing policies or a predictor of the effects of new policies, as I have argued elsewhere in the context of tax compliance (see e.g., Graetz, Reinganum and Wilde, 1986). From a positive point of view, precommitment is a very dubious assumption. Furthermore, even if the social authorities can precommit to an overall allocation of resources to apprehension, law enforcement agencies have a significant degree of control over how those resources translate into apprehension rates for specific crimes. In other words, we need a theory of how $g_1(r)$ and $g_2(r)$, to use the notation of this paper, are generated.

We also need a better understanding of the degree to which precommitment to a given sanctions policy is possible. Judges and juries, for example, have a great deal of control over sanctions, and it is difficult to bind these agents of society to prespecified sanctions policies. If one moves out of the Becker Tradition in this respect, however, motives other than deterrence must come into play (Reinganum and Wilde, 1986). In this case, even if the choice of sanctions is based entirely on considerations other than deterrence, the existence of positive sanction levels will imply some deterrent effect anyway. That a theory based only on deterrence can explain even a part of the criminal justice system may therefore be purely spurious.

APPENDIX

Proof of Proposition 1:

Taking derivatives of $W(s_1, s_2, r)$ gives

$$W_{s_1} = -p_1 \int_0^{p_2 s_2} (p_1 s_1 - h_1 - \sigma p_1 s_1) f(p_1 s_1, b_2) db_2 \quad (A1)$$

$$- p_1 \sigma \int_{p_1 s_1}^{\infty} \int_0^{b_1 - p_1 s_1 + p_2 s_2} f(b_1, b_2) db_2 db_1$$

$$- p_1 \int_{p_1 s_1}^{\infty} (b_1 - h_1 - \sigma p_1 s_1) f(b_1, b_1 - p_1 s_1 + p_2 s_2) db_1$$

$$+ p_1 \int_{p_2 s_2}^{\infty} (b_2 - h_2 - \sigma p_2 s_2) f(b_2 - p_2 s_2 + p_1 s_1, b_2) db_2,$$

$$W_{s_2} = -p_2 \int_0^{p_1 s_1} (p_2 s_2 - h_2 - \sigma p_2 s_2) f(b_1, p_2 s_2) db_1 \quad (A2)$$

$$- p_2 \sigma \int_{p_2 s_2}^{\infty} \int_0^{b_2 - p_2 s_2 + p_1 s_1} f(b_1, b_2) db_1 db_2$$

$$- p_2 \int_{p_2 s_2}^{\infty} (b_2 - h_2 - \sigma p_2 s_2) f(b_2 - p_2 s_2 + p_1 s_1, b_2) db_2$$

$$+ p_2 \int_{p_1 s_1}^{\infty} (b_1 - h_1 - \sigma p_1 s_1) f(b_1, b_1 - p_1 s_1 + p_2 s_2) db_1,$$

and

$$W_r = (W_{s_1} s_1 g_1' / g_1) + (W_{s_2} s_2 g_2' / g_2) - 1. \quad (A3)$$

part (a): Clearly, from (A3), it cannot be the case that both s_1^* and s_2^* are interior to $[0, \bar{s}]$, at least one must equal \bar{s} or otherwise $W_r = -1$ which cannot be part of an optimum (since it implies $r^* = 0$). It is possible, however that both $s_1^* = \bar{s}$ and $s_2^* = \bar{s}$.

part (b): The first order conditions for s_1^* and s_2^* are symmetric so suppose, without loss of generality, that $s_1^* \in (0, 1)$ and $s_2^* = \bar{s}$. Differentiating $W_{s_1} = 0$ totally with respect to s_1 and h_1 gives

$$(\partial W_{s_1} / \partial s_1) ds_1 + \left[\int_0^{p_2 s_2} f(p_1 s_1, b_2) db_2 + \int_{p_1 s_1}^{\infty} f(b_1, b_1 - p_1 s_1 + p_2 s_2) db_1 \right] dh_1 = 0. \quad (A4)$$

Second-order-conditions (which I assume to hold) require $\partial W_{s_1}/\partial s_1 < 0$. The term in brackets is positive. Thus $ds_1^*/dh_1 > 0$ if $s_1^* \in (0, \bar{s})$. Differentiating $W_{s_1} = 0$ totally with respect to s_1 and h_2 gives

$$(\partial W_{s_1}/\partial s_1) ds_1 - \left[\int_{p_2 s_2}^{\infty} f(b_2 - p_2 s_2 + p_1 s_1, b_2) \right] dh_2 = 0. \quad (A5)$$

Thus it is clearly the case that $ds_1^*/dh_2 < 0$ if $s_1^* \in (0, \bar{s})$.

part (c): Suppose again without loss of generality that $s_1^* \in (0, \bar{s})$ and $s_2^* = \bar{s}$. Since $s_2^* = \bar{s}$, W_{s_2} is irrelevant. Thus, to evaluate ds_1^*/dh_1 and dr^*/dh_1 we need to consider (A1) and (A3) only.

Differentiating $W_{s_1} = 0$ and $W_r = 0$ totally with respect to s_1 , r , and h_1 gives

$$\begin{bmatrix} W_{s_1 s_1} & W_{s_1 r} \\ W_{s_2 s_1} s_2 g_2' & \phi(s_1, r, h_1) \end{bmatrix} \begin{bmatrix} ds_1^*/dh_1 \\ dr^*/dh_1 \end{bmatrix} = \begin{bmatrix} -W_{s_1 h_1} \\ -W_{s_2 h_1} s_2 g_2' \end{bmatrix}, \quad (A6)$$

where double subscripts denote the obvious second partial derivatives, and

$$\phi(s_1, r, h_1) = W_{s_2 r} s_2 g_2' + W_{s_2 s_2} s_2 g_2'' - g_2''. \quad (A7)$$

The proof of part (b) of this proposition has shown that $W_{s_1 h_1} > 0$ and $W_{s_2 h_1} < 0$.

Second-order-conditions require that $W_{s_1 s_1} < 0$, $\phi(s_1, r, h_1) < 0$ and

$W_{s_1 s_1} \phi(s_1, r, h_1) - W_{s_2 s_1} s_2 g_2' W_{s_1 r} > 0$. Thus $ds_1^*/dh_1 > 0$ unambiguously only if $W_{s_1 r} < 0$. A similar result obtains for ds_1^*/dh_2 : the sign of $W_{s_1 r}$ is crucial.

Q.E.D.

Proof of Proposition 2:

Social welfare is essentially as given in equation (2) except $p_1 = g_1(r_1)$, $p_2 = g_2(r_2)$ and $r = r_1 + r_2$. Thus W_{s_1} and W_{s_2} are identical to (A1) and (A2) respectively. But now (A3) is replaced by

$$W_{r_1} = W_{s_1} s_1 g_1' / g_1 - 1 \quad (A8)$$

and

$$W_{r_2} = W_{s_2} s_2 g_2' / g_2 - 1. \quad (A9)$$

Clearly if $s_1 \in (0, \bar{s})$ or $s_2 \in (0, \bar{s})$ then $r_1^* = 0$ or $r_2^* = 0$ respectively (since $W_{s_i} = 0$ implies $W_{r_i} = -1$ for $i = 1, 2$). Hence it must be that $W_{s_i} > 0$ and $s_i^* = \bar{s}$ for $i = 1, 2$. This proves part (a).

The proof of part (b) is identical to that given for part (b) of Proposition 1.

Q.E.D.

Proof of Proposition 3:

By the hypothesis of the proposition, $b_1 = b = b_2$, $h_2 > h_1$, and $g_2(r) < g_1(r)$ for all $r > 0$. In this case, since the private benefits are the same for both crimes,

$$W(s_1, s_2, r) = \begin{cases} \int_{e_2}^{\infty} (b - h_2 - \sigma e_2) f(b) db - r & \text{if } e_1 > e_2 \\ \int_{e_1}^{\infty} (b - h_1 - \sigma e_1) f(b) db - r & \text{if } e_1 \leq e_2 \end{cases} \quad (\text{A10})$$

where $e_i = g_i(r)s_i$ is the expected sanction on a crime of type i , and $f(\cdot)$ is the density of benefits. Note that indifference is resolved in favor of the less harmful act.

Claim 1: $s_1^* = \bar{s}$ implies $s_2^* = \bar{s}$.

Proof of claim: Suppose $s_1^* = \bar{s}$. Then since $g_1(r) > g_2(r)$ for all $r > 0$, $g_1(r)\bar{s} = g_1(r)s_1^* > g_2(r)s$ for all $s \in [0, \bar{s}]$ and $r > 0$. Hence $g_1(r^*)\bar{s} > g_2(r^*)s_2^*$, or $e_1^* > e_2^*$, and W is given by the first branch of (A10). But the standard argument used in this paper now implies $s_2^* = \bar{s}$.

Claim 2: $s_2^* = \bar{s}$ implies $s_1^* = \bar{s}$.

Proof of claim: Consider $W(s_1, \bar{s}, r)$ for r fixed. Define $\bar{e}_2(r) = g_2(r)\bar{s}$. Then W will be maximized at some $s_1 = \hat{s}_1(r) \leq \bar{e}_2(r)/g_1(r)$ since for $s_1 > \bar{e}_2(r)/g_1(r)$ W is given by the first branch of (A10), which is independent of s_1 . As s_1 falls to $\bar{e}_2(r)/g_1(r)$ there is a discontinuous jump upwards in welfare since the offender shifts to committing the first crime. Since $\bar{e}_2(r)/g_1(r) < \bar{s}$ for all r — $g_2(r) < g_1(r)$ for all r —it is certainly true at the optimal r that $s_1^* = \hat{s}_1(r^*) \leq \bar{e}_2(r^*)/g_1(r^*) < \bar{s}$.

Together claims 1 and 2 imply $s_1^* = \bar{s}$ is impossible. Hence $s_2^* = \bar{s}$ (since one sanction must equal \bar{s}) and $s_1^* < \bar{s}$. This proves part (a). But one can say more.

Claim 3: $e_1^* = e_2^*$; i.e., $s_1^* = g_2(r^*)\bar{s}/g_1(r^*)$.

Proof of claim: Suppose $e_1^* < e_2^*$. Then W is given by the second branch (A10). Thus it is possible to lower r^* slightly, raise s_1^* slightly and leave $g_1(r^*)s_1^* = e_1^*$ unchanged. If r^* is lowered, $g_2(r^*)$ falls, reducing $e_2^* = g_2(r^*)\bar{s}$. But, since $e_1^* > e_2^*$, a small enough reduction in r^* will keep W on the second branch of (A10). Since e_1^* is unchanged but r^* falls, W must rise. Therefore $e_1^* = e_2^*$ since $e_2^* > e_1^*$ cannot be optimal.

Given Claims 1-3, the social authority's problem can be rewritten as

$$\max_{r, s_1} \int_{g_1(r)s_1}^{\infty} [b - h_1 - \sigma g_1(r)s_1] f(b) db - r \quad (\text{A11})$$

subject to $g_1(r)s_1 = g_2(r)\bar{s}$.

Let λ be the multiplier associate with the constraint. Then first order conditions can be written as

$$-[g_1(r)s_1 - h_1 - \sigma g_1(r)s_1] f(g_1(r)s_1) g_1(r) = \int_{g_1(r)s_1}^{\infty} \sigma g_1(r) f(b) db + \lambda g_1(r), \quad (\text{A12})$$

and

$$\begin{aligned} & -[g_1(r)s_1 - h_1 - \sigma g_1(r)s_1] f(g_1(r)s_1) g_1'(r) - \int_{g_1(r)s_1}^{\infty} \sigma g_1'(r)s_1 f(b) db = \\ & 1 - \lambda[g_1'(r)s_1 - g_2'(r)\bar{s}]. \end{aligned} \quad (\text{A13})$$

Solving (A12) and (A13) for λ gives $\lambda = -1/g_2'(r)\bar{s}$. Thus, r^* and s_1^* are given by

$$-[g_1(r)s_1 - h_1 - \sigma g_1(r)s_1] - \int_{g_1(r)s_1}^{\infty} \sigma f(b) db = 1/g_2'(r)\bar{s} \quad (\text{A14})$$

and

$$g_1(r)s_1 = g_2(r)\bar{s}. \quad (\text{A15})$$

substituting (A15) into (A14) gives a single equation for r^* :

$$-[g_2(r^*)\bar{s} - h_1 - \sigma g_2(r^*)\bar{s}] - \int_{g_2(r^*)\bar{s}}^{\infty} \sigma f(b) db = 1/g_2'(r^*)\bar{s}. \quad (\text{A16})$$

Differentiating (A16) totally with respect to r^* and h_1 gives, by second-order-conditions associated with r^* , that $dr^*/dh_1 > 0$. Thus, from (A15)

$$\frac{ds_1^*}{dh_1} = \bar{s} \frac{d[g_2(r^*)/g_1(r^*)]}{dr} (dr^*/dh). \quad (\text{A17})$$

Since $dr^*/dh > 0$, the sign of ds_1^*/dh_1 equals the sign of $d[g_2(r^*)/g_1(r^*)]/dr$. This completes the proof of part (b).

Q.E.D.

Proof of Proposition 4:

Using the behavioral rule specified in (1), $b_2 = \alpha b_1$ implies that

$$W(s_1, s_2, r) = \int_{p, s_1}^{\max\{p(s_2 - s_1)/(\alpha - 1), ps_1\}} (b_1 - h_1 - \sigma ps_1) f(b_1) db_1 \quad (A18)$$

$$+ \int_{\max\{ps_2/\alpha, p(s_2 - s_1)/(\alpha - 1)\}}^{\infty} (\alpha b_1 - h_2 - \sigma ps_2) f(b_1) db_1 - r,$$

where $g_1(r) = p = g_2(r)$ is the common probability of apprehension. The basic argument of this paper implies that $s_1^* = \bar{s}$, $s_2^* = \bar{s}$, or both. Suppose $s_1^* = \bar{s}$. Then $\max\{p(s_2 - s_1)/(\alpha - 1), ps_1\} = ps_1$ and $\max\{ps_2/\alpha, p(s_2 - s_1)/(\alpha - 1)\} = ps_2/\alpha$. Hence

$$W(s_1, s_2, r) = \int_{ps_2/\alpha}^{\infty} (\alpha b_1 - h_2 - \sigma ps_2) f(b_1) db_1 - r, \quad (A19)$$

so that

$$W_r = (p' W_{s_2} s_2 / p) - 1. \quad (A20)$$

Clearly, $W_{s_2} = 0$ is impossible so $s_2^* = \bar{s}$ as well.

Suppose, then, that $s_2^* = \bar{s}$. If $\max\{p(s_2 - s_1)/(\alpha - 1), ps_1\} = ps_1$, then it remains the case that only crime 2 is committed, so that s_1 is irrelevant. This condition reduces to $s_1 > \bar{s}/\alpha$. Thus, one needs only consider maximizing W with respect to $s_1 \in [0, \bar{s}/\alpha]$. If $s_1^* = \bar{s}/\alpha$ on this range, then only crime 2 is committed so any $s_1 \in [\bar{s}/\alpha, \bar{s}]$ is optimal. The reformulated social problem is

$$\max_{\substack{r \geq 0 \\ 0 \leq s_1 \leq \bar{s}/\alpha}} \int_{ps_1}^{p(\bar{s} - s_1)/(\alpha - 1)} (b_1 - h_1 - \sigma ps_1) f(b_1) db_1 \quad (A21)$$

$$+ \int_{p(\bar{s} - s_1)/(\alpha - 1)}^{\infty} (\alpha b_1 - h_2 - \sigma p\bar{s}) f(b_1) db_1.$$

In this case,

$$W_{s_1} = - \left[\frac{p(\bar{s} - s_1)}{\alpha - 1} - h_1 - \sigma ps_1 \right] f \left[\frac{p(\bar{s} - s_1)}{\alpha - 1} \right] \left[\frac{p}{\alpha - 1} \right] \quad (A22)$$

$$- (ps_1 - h_1 - \sigma ps_1) f(ps_1) p + \sigma p \int_{ps_1}^{p(\bar{s} - s_1)/(\alpha - 1)} f(b_1) db_1$$

$$+ \left[\frac{\alpha p(\bar{s} - s_1)}{\alpha - 1} - h_2 - \sigma p\bar{s} \right] f \left[\frac{p(\bar{s} - s_1)}{\alpha - 1} \right] \left[\frac{p}{\alpha - 1} \right],$$

and

$$W_{s_1}(s_1 = \bar{s}/\alpha) = [p/(\alpha - 1)](\alpha h_1 - h_2)f'(\bar{s}/\alpha).$$

Thus $W_{s_1}(s_1 = \bar{s}/\alpha) < 0$ if and only if $\alpha h_1 < h_2$.

Q.E.D.

FOOTNOTES

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- 1. The notation follows Shavell (1985) where possible. In that paper Shavell includes a weighting factor, denoted by β , which translates private benefits into social welfare. In this paper we assume, without loss of generality, that $\beta = 1$.
- 2. It is always possible that it is socially optimal to devote no resources whatsoever to apprehension. This could happen for example, if for $i = 1, 2$, h_i is close to zero, $g_i(0) = 0$, and $g_i'(0)$ is sufficiently large. I will not formally specify sufficient conditions for $r^* > 0$ but instead will simply assume such conditions hold.
- 3. As with r^* , I will simply assume sufficient conditions for $r_1^* > 0$ and r_2^* hold (see footnote 2).
- 4. There is no analogue to Proposition 1, part c, because both s_1^* and s_2^* equal \bar{s} .
- 5. See Shavell (1989a) for a subsequent analysis of this point in the context of a single-crime model. In the language of that paper, the distinction is between *specific* enforcement and *general* enforcement.

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